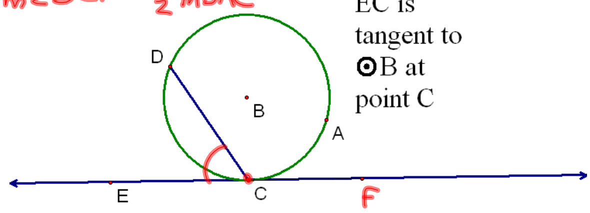
10-5 Apply Other Angle Relationships in Circles

Theorem 10.11--If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

$m\angle DCE = \frac{1}{2} m\widehat{DC}$

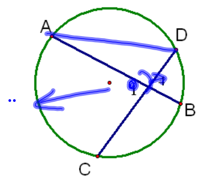
$m\angle DCF = \frac{1}{2} m\widehat{DAC}$



$\overleftrightarrow{EC}$ is tangent to ⊙B at point C

Given: picture

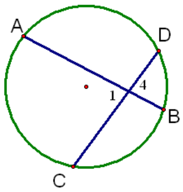
Prove: $m\angle 4 = \frac{1}{2}(m\widehat{AC} + m\widehat{DB})$



S.	R.
① ~	① Given
② Draw $\overline{AD}$	② Through any 2 pts there is exactly 1 line
③ $m\angle 4 = m\angle A + m\angle D$	③ Ext < of Δ thm
④ $m\angle A = \frac{1}{2} m\widehat{DB}$ $m\angle D = \frac{1}{2} m\widehat{AC}$	④ Measure of Inscribed < Thm
⑤ $m\angle 4 = \frac{1}{2} m\widehat{DB} + \frac{1}{2} m\widehat{AC}$	⑤ Subst
⑥ $m\angle 4 = \frac{1}{2}(m\widehat{DB} + m\widehat{AC})$	⑥ Subst
⑦ $m\angle 4 = \frac{1}{2}(m\widehat{AC} + m\widehat{DB})$	⑦ Commutative

Theorem 10-12--Angles Inside the Circle Theorem--The measure of an angle formed on the inside of a circle (by 2 secants or 2 chords) is half the sum of the measures of the intercepted arcs.

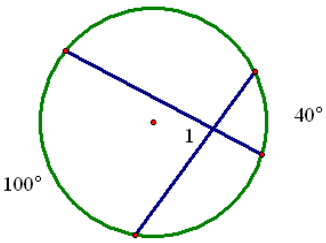
$m\angle 4 = \frac{1}{2}(m\widehat{AC} + m\widehat{DB})$



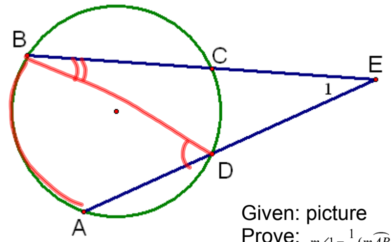
$m\,\text{inside} = \frac{1}{2}(\text{sum arcs})$

gsp

Find the measure of the angle.



$m\angle 1 = \frac{1}{2}(100 + 40)$
$= 70°$



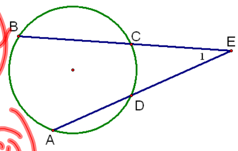
Given: picture

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{AB} - m\widehat{DC})$

① ~	① Given
② Draw $\overline{BD}$	② Through any 2 pts ...
③ $m\angle BDA = m\angle DBC + m\angle 1$	③ Ext < of Δ thm
④ $m\angle BDA - m\angle DBC = m\angle 1$	④ Subtr
⑤ $m\angle 1 = m\angle BDA - m\angle DBC$	⑤ Symm.
⑥ $m\angle DBC = \frac{1}{2} m\widehat{DC}$ $m\angle BDA = \frac{1}{2} m\widehat{AB}$	⑥ Measure of Inscribed < Thm
⑦ $m\angle 1 = \frac{1}{2} m\widehat{AB} - \frac{1}{2} m\widehat{DC}$	⑦ Subst
⑧ $m\angle 1 = \frac{1}{2}(m\widehat{AB} - m\widehat{DC})$	⑧ Subst

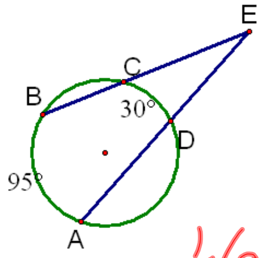
Theorem 10-13--Angles Outside the Circle Theorem--The measure of an angle formed on the outside of a circle (by 2 secants, 2 tangents, or secant and a tangent) is half the difference of the measures of the intercepted arcs.

outside = $\frac{1}{2}$(difference of arcs)



$m\angle 1 = \frac{1}{2}(m\widehat{AB} - m\widehat{CD})$

gsp

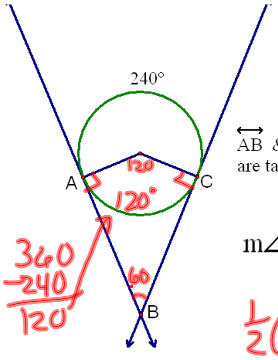


m∠E =

$\frac{1}{2}(95-30)$

65

32.5°



$\overleftrightarrow{AB}$ & $\overleftrightarrow{BC}$ are tangents

m∠B = 60°

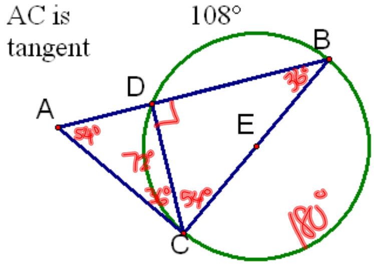
360
−240
120

$\frac{1}{2}(240-120)$

120

60°

$\overline{AC}$ is tangent

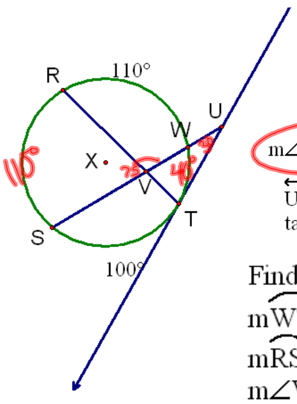


BC is the diam

$m\angle A = \frac{1}{2}(180-72) = 54°$

$m\angle B = \frac{1}{2}72° = 36°$

$m\angle DCA$ =



m∠WUT = 30°

$\overleftrightarrow{UT}$ IS tangent

Find

$m\widehat{WT}$ = 40°

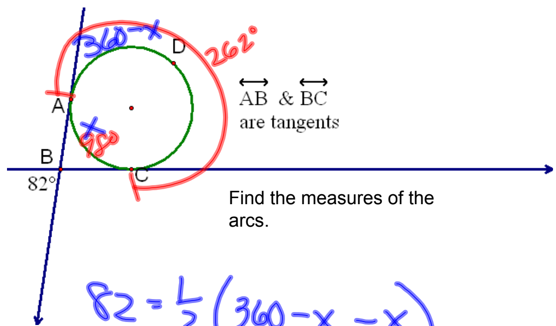
$m\widehat{RS}$ = 110°

m∠WVT = $\frac{1}{2}(110+40) = 75°$

m∠RVW = 105°

$30 = \frac{1}{2}(180 - m\widehat{WT})$

$60 = (180 - m\widehat{WT})$



$\overleftrightarrow{AB}$ & $\overleftrightarrow{BC}$ are tangents

Find the measures of the arcs.

$82 = \frac{1}{2}(360 - x - x)$

$(360-2x)$

$82 = 180 - x$

$98° = x$

HW
p683-685
#s 3-11, 16a, 17,
18, 20, 21

Attachments

10_6_gsp_example.gsp